

## DESIGN OF PSEUDO-QUADRATURE MIRROR FILTER BANK FOR HIGH QUALITY SUBBAND CODING

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**ABSTRACT** - An approximation method is presented to design pseudo-QMF filter banks for high quality subband coding. This method takes into account the implementation structure of the whole subband system.

### 1. INTRODUCTION

Subband coding systems are nowadays widely used for sound (Dehery 1991, Smyth & al 1988, Theile & al 1988) and speech (CCITT G722 Norm, Esteban & al 1987) processing. They indeed allow a significant bit-rate reduction without altering the quality of the processed signal.

A subband coding system can schematically be divided into three main phases: the analysis, the subband coding-decoding and the synthesis. The analysis consists in splitting the input signal in a certain number of signals each of which representing a determined part of the original signal spectrum and which are called "subbands". This operation is done, in principle, by a band-pass filter bank which isolates the different spectrum parts, followed by a decimation process. At this step, the subband coding-decoding allows to reduce the bit-rate needed to store or transmit the signal. Finally, the synthesis reconstructs the signal from the subbands. It is the dual operation of the analysis. It consists in an interpolation process followed by a band-pass filter bank.

The analysis and the synthesis are obviously elaborated to preserve the original signal if there is no subband coding-decoding. This implies that "aliasing" and "imaging" effects, generated respectively by the decimation and the interpolation processes, must be eliminated.

The first solution given to this problem was made by Galand with the so-called Quadrature Mirror Filters (QMF) (Galand 1983). Indeed, those filters yield a reconstructed signal free from "aliasing" and "imaging" components. For that reason, they have been widely used. However, their main drawback was the important required complexity.

Nowadays, the best solution is achieved with pseudo-QMF filters (Nussbaumer 1981). Those filters surpass the QMF performances, specially concerning the required complexity (Masson & al 1985, Mau 1991). Moreover, pseudo-QMF filters induce a processing delay much smaller, which is a great advantage for transmission applications.

Our aim is to present an original approximation method for the design of the pseudo-QMF filter banks which minimises the "aliasing" and "imaging" effects and is suited for high quality subband coders. In section 2, we present the pseudo-QMF filter banks and their structure of implementation. The next section describes the classical approximation method. We then show some reconstruction insufficiencies with filter banks obtained by such method. In the section 4, we present a new approximation program, which combined with the classical one, yields better reconstruction results. Finally, we give some conclusions.

### 2. PRESENTATION OF THE PSEUDO-QMF FILTER BANK

Pseudo-QMF filter banks are defined by a linear phase prototype filter and by the number M of created subbands. The M analysis filters  $h_k(n)$  and synthesis filters  $f_k(n)$  ( $k=0, 1, \dots, M-1$ ) are obtained by modulating the coefficient of a prototype filter  $h(n)$  by "cosine" functions defined as followed :

N = Length of the prototype filter

$$h_k(n) = h(n) A_k(n) \quad (1)$$

$$f_k(n) = h(n) S_k(n) \quad (2)$$

$$A_k(n) = 2 \cos(\pi(2k+1)(n-(N-1)/2)/2M-\theta(k)) \quad (3)$$

$$S_k(n) = 2 \cos(\pi(2k+1)(n-(N-1)/2)/2M+\theta(k)) \quad (4)$$

Several definitions of the  $\Theta(k)$  term allow to obtain the required characteristics of the subband system (Cox 1986). We will only retain the definition given by relation (5), as it leads to an efficient implementation structure (Masson & al 1985, Mau 1991).

$$\Theta(k) = (2k+1)\pi/4 \quad (5)$$

In order to eliminate the parasitic "aliasing" and "imaging" effects, it can be shown that the prototype filter should satisfy the following relations :

$$\begin{aligned} \phi &= 2\pi f/f_E \\ A(\phi) &= |H(e^{j\phi})| \\ A^2(\phi) + A^2(\pi/M-\phi) &= 1 & 0 < \phi < \pi/M \\ A(\phi) &= 0 & \pi/M < \phi < \pi \end{aligned} \quad (6)$$

$$(7)$$

In practice, it is impossible to strictly satisfy those relations and thus "aliasing" and "imaging" components are not perfectly removed. Thus, the aim of the approximation program is to minimise these last ones.

The implementation of pseudo-QMF filter banks can be efficiently realised by the use of discrete cosine transform (DCT). We will consider in this paper only prototype filter which has an odd length. In that case, the analysis filter bank can be implemented as shown at figure 1 (Mau 1991).

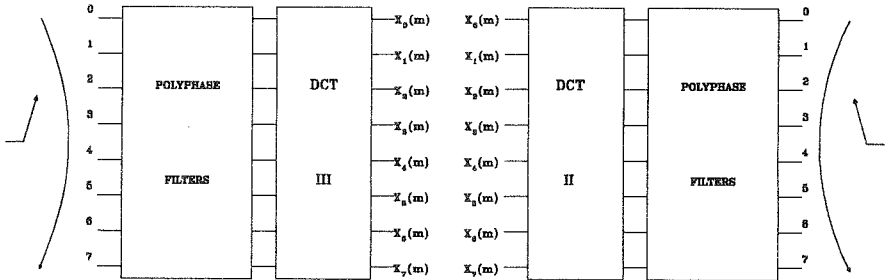


figure 1

In figure 1, the rotating arrow symbolises the division of the input signal into successive blocks of M samples. The block "polyphase filters" is constituted by M short length filters of which coefficients are directly issued from the prototype filter ones. Next, the output values of this block are processed by a DCT-III which yields the subband samples  $X_k(m)$ .

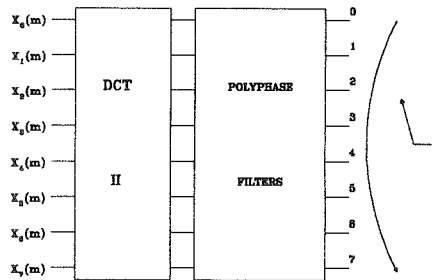


figure 2

In a dual form, figure 2 presents the implementation of the synthesis filter bank. The subband samples are treated by a DCT-II which is in occurrence the inverse transform of the DCT-III. Resulting values are then processed by a second set of short length polyphase filters whose the coefficients are also directly issued from the prototype filter coefficients.

In practice, the subband system is implemented on a DSP. In order to optimise the use of the DSP by the realisation of a very symmetric implementation of the polyphase filter blocks, it can be shown (Renard 1992) that the length of the prototype filter should satisfy one of the two following relations :

$$\begin{aligned} N &= 4 \kappa M - M + 1 & \kappa & \text{integer} & (8) \\ \text{or } N &= 4 \kappa M + M - 1 & \kappa & \text{integer} & (9) \end{aligned}$$

### 3. CLASSICAL APPROXIMATION OF THE PROTOTYPE FILTER

As the coefficients of the analysis and synthesis polyphase filters are extracted from the prototype filter ones, the design of the filter bank can be based on the approximation of either the polyphase

filters or the prototype filter.

In general, the last possibility is chosen and coefficients are obtained by a gradient method ("steepest descent") where cost function C to be minimised, is derived from relations (6) and (7) :

$$C = \frac{1}{N_R} \sum_{i=0}^{N_R-1} \left( 1 - A^2 \left( \frac{i\pi}{N_R M} \right) - A^2 \left( \frac{\pi}{M} - \frac{i\pi}{N_R M} \right) \right)^2 + \alpha \frac{1}{N_A} \sum_{j=0}^{N_A-1} A^2 \left( \frac{\pi}{M} + \left( \pi - \frac{\pi}{M} \right) \frac{j}{N_A} \right) \quad (10)$$

where  $N_R$  are the number of points of discretisation of relation (8)

$N_A$  are the number of points of discretisation of relation (9)

The parameter  $\alpha$  is introduced to balance the relative importance of the two terms of relation (10).

We have elaborated a program on a PC which minimises the cost function C with the help of a gradient method. In order to limit the number of input data, we have fixed the parameters N and M as follows :

M = 16

N = 177

Note that N satisfies relation (8).

The parameters of the program are the following :

- starting set of coefficients. This set cannot be taken by hazard, otherwise the program may diverge. In fact, the starting set must lead to a filter with low-pass characteristics. A Hanning filter seems to be a good choice.

- $N_A$  and  $N_R$  should be as high as possible. However, in order to limit the computation time, we have chosen :

$N_R = 100$

$N_A = 800$

- $\alpha$

- gradient step. The best experimental value has been  $10^{-5}$ .

- lim : a limit of convergence is taken into account in order to stop the program if the variation of cost function is smaller than that limit.

- itmax : a maximum number of iteration has been introduced to halt the program. It thus allows to change the value of the other parameters and the starting set of coefficients.

By successive trials, we have obtained a prototype filter, which at first glance, has proper characteristics for subband coding (Cf. figure 3).

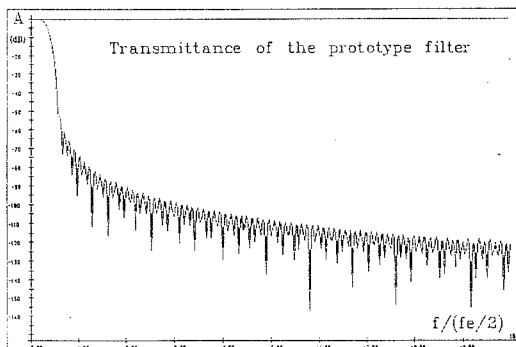


figure 3

So to verify the reconstruction performances of the system implemented with this prototype filter, a test program has been elaborated on a PC which simulates the implementation structure of figures 1 and 2 and computes the error signal introduced by the pseudo-QMF system. Many experiments were made with different input signals and put reconstruction problems into evidence. For example, if the input signal is a sinusoid of amplitude 30000 and frequency 1000 Hz (sampling frequency is 44.1 kHz), the error signal is shown in figure 4 where the first window represents 4096 samples of the error signal, window "zoom 1", 512 samples taken in the previous signal and the last window "spectre 1", the logarithmic gain of the DFT (Discrete Fourier Transform) of the signal presented in window "zoom 1".

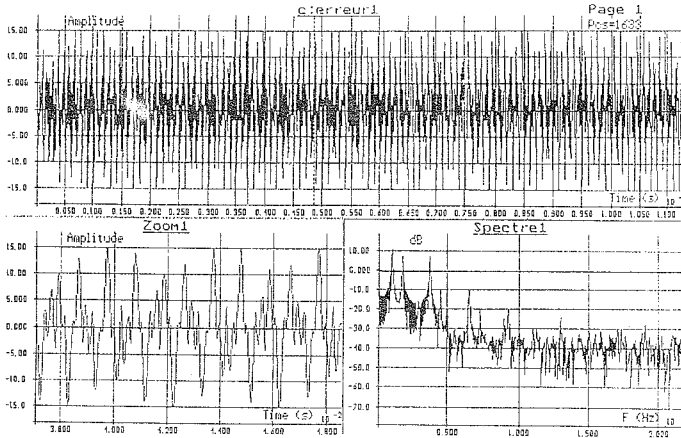


figure 4

In the window "spectre 1", we observe the presence of several peaks. The peak of frequency 1000 Hz is obviously coming from the input signal. However, all the other peaks are explainable only in terms of not sufficiently attenuated "aliasing" and "imaging" components. Those components may become audible, which cannot be tolerated for high quality subband coding.

We have then tried to design a new set of coefficients which lead to a subband system where the default previously described are reduced. This method is described in the next section.

#### 4. TEMPORAL APPROXIMATION OF THE PROTOTYPE FILTER

To obtain better reconstruction performances, we need to introduce new relations for the approximation. These new relations are established in the time domain. Indeed, in order to reconstruct a signal identical to the input signal with a delay, the impulse response of the global subband system should be the Dirac impulse response.

Nevertheless, the impulse response of the pseudo-QMF system can be derived from the implementation structure of figure 1 and 2. As the coding-decoding phase is not considered in this study, the DCT-III and DCT-II are put back to back. But, these last transforms are inverse of each other, and can thus be removed to lead to a simplified structure where they do not appear. For that simplified structure, the impulse response is dependant of the solicitation time. Indeed, in function of the time, only one of the polyphase filters is fed with a non zero value and thus it exists M different impulse response. Their global shape is constituted by a succession of peaks. The approximation problem is now equivalent to find a prototype filter coefficients set that reduce as much as possible the amplitude of the peaks excepted the central one which must be equal to unity.

Each of the amplitude of the peaks can be expressed as a function of the prototype filter coefficients. We note them as follows :

$p_{s,n}$  : amplitude of the  $n^{\text{th}}$  peak of  $s^{\text{th}}$  impulse response.

Those constraints can be thus expressed, in order to fit a steepest descent algorithm, with the following relations :

$$\text{Constraints on the central peaks : } (1-p_{s,n})^2 = 0 \quad (11)$$

$$\text{Constraints on the other peaks : } p_{s,n}^2 = 0 \quad (12)$$

We have developed a new approximation program based only on these last constraints. The input parameters of this program are :

- starting coefficients set;
- $\alpha_n$  : allow to weight the different constraints;
- gradient step;
- lim;
- itmax.

The different trial have shown that this program can effectively minimise constraints (11) and (12). However, this optimisation of the impulse response leads to a worse rejection of the prototype filter. In some case, the minimal attenuation reaches 45 dB which is very inefficient for subband coding (Dehery 1991). For that reason, we have decided to combine both program to obtain better results than those shown in the windows of figure 4. The resulting transmittance, compared to the classical one, is presented in figure 5.

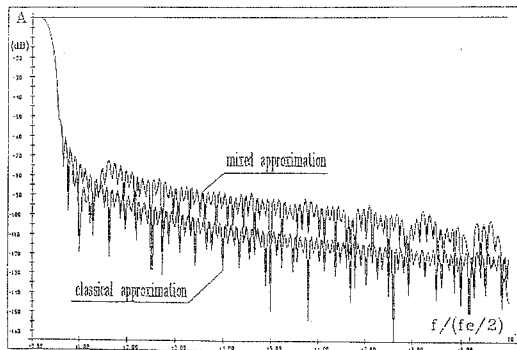


figure 5

We note that the rejection of the new prototype filter is lower excepted in the low frequency region. We have then proceeded to the simulation test with a sinusoid of 1000 Hz to determine the new error signal. Results are presented at figure 6. The windows represent the following curves :

Window "erreur 1" : 4096 samples of figure 3

Window "erreur 2" : 4096 samples of the error signal obtained with the new prototype filter.

Window "zoom 1" : 512 samples of window "erreur 1"

Window "zoom 2" : 512 samples of window "erreur 2"

Window "spectre 1" : logarithmic gain of the signal in window "zoom 1"

Window "spectre 2" : logarithmic gain of the signal in window "zoom 2"

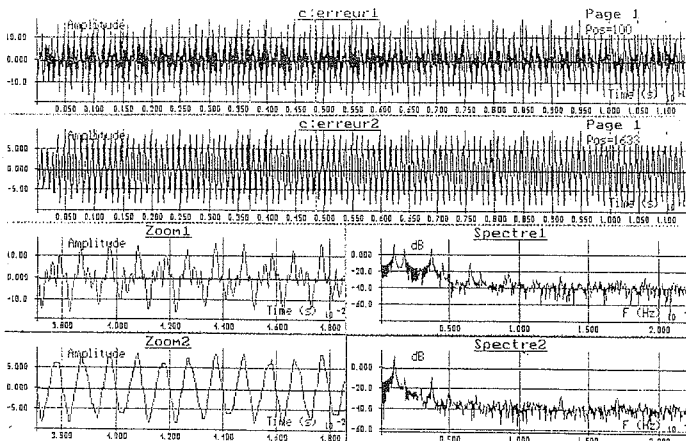


figure 6

We can see that the magnitude of the error signal in window "zoom 1" is bigger than the new error signal. Moreover, in window "spectre 2", we see that there is only one important peak. The others have been strongly attenuated. Comparison made on other test signal lead to the same conclusions.

For these reasons, we can say that we improve the reconstruction performances of the pseudo-QMF system by introducing new constraints taking into account its implementation structure.

## 5. CONCLUSIONS

We have show that the classical approximation of a pseudo-QMF prototype filter can lead to some reconstruction insufficiencies for the whole system. A solution consists to set new constraints which take into account the implementation structure of the system itself and which deal with the impulse response of the global system.

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